

Transmit Strategies for Massive Machine-Type Communications based on Mean Field Games

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Abstract—Massive Machine Type Communications are one of the three main type of communication applications in upcoming 5G wireless networks. In this type of communication, the network is required to handle a huge number of devices transmitting information to the same base station receiver in an uncoordinated manner. In this setting, the problem of minimizing energy usage while achieving QoS requirements is a very complex stochastic control problem with a very large number of optimizing agents. In this paper, we propose a Mean Field Games model for this problem that reduces the complexity by a great deal and is thus amenable to numerical solution. Our model is general enough to include generic rate functions, arbitrary energy and QoS requirements per user, different channel fading models, and design knobs for determining the importance of different performance goals. We provide details of the proposed numerical solution and present numerical results that illustrate the characteristics of the obtained control policy.

Index Terms—Massive Machine Type Communications, Energy Efficiency, Quality of Service, Mean Field Games.

I. INTRODUCTION

With the proliferation of Internet of Things (IoT), one of the key requirements for wireless networks of the future will be their ability to serve a huge number of wireless devices. It is foreseen that in 5G (and beyond) networks, the density of Machine-Type Communications (MTC) networks may surpass 1 million of devices per km² [1]. Thus, despite network densification, an extremely large number of machines will desire to communicate with a serving base station (BS) each with a small traffic requirement. Despite the low traffic per device, the massive MTC (mMTC) scenario is one of the most challenging of 5G and beyond, since we desire a communication protocol that at the same time is (i) decentralized (overheads for millions of devices cannot be tolerated), (ii) energy-efficient (machines may need to be unattended for long time), and (iii) guarantees QoS performance (such as low average service time).

A typical barrier in this case is the *curse of dimensionality*, whereby the large system state-space makes the discovery of the optimal protocol extremely challenging and computationally infeasible. To overcome this problem, and transform the dimensionality from a curse into a blessing, we propose a novel model based on Mean Field Games (MFG) [2] [3] [4]. In MFG, the optimal action of each device depends only on the average behavior of other devices, which allows to efficiently compute the optimal solution.

More specifically, we focus on the mMTC uplink of a single cell containing one BS and a very large number of transmitting devices using a Non-Orthogonal Medium Access (NOMA) scheme which has been proposed as a candidate for 5G wireless networks mMTC. This type of medium access is basically a Code Division Multiple Access (CDMA) scheme with the difference that the transmitting devices do not have codes that are completely orthogonal to each other. Although orthogonal codes would be desirable for nearby devices, offline assignment of orthogonal codes would require an impossibly large number of orthogonal codes, while online assignment would require a high degree of coordination, which is also prohibitive in our setting.

NOMA resolves this issue by an a priori assignment of codes that are quasi-orthogonal thus requiring a reduced number of codes to serve all devices, allowing some small degree of overlap due to randomness. Yet, when any subset of the devices transmit simultaneously, the transmitted information can still be decoded at the receiver, albeit at a lower data rate than if all devices transmitted with orthogonal codes.

In addition, NOMA can be combined with transmit power control to improve both throughput and energy efficiency. Power control has always been recognized as an important problem for multiuser communications. In particular distributed power control policies, where mobile terminals can freely choose their transmit power level $p_i(t)$ and do not need to be controlled from central nodes, are of special importance as they avoid the complexity, signaling overhead and delay of centralized solutions. Notice that this is a very general scenario where for example $p_i(t)$ can be chosen between two extremes to emulate TDMA, or with a probability to emulate CSMA.

In our model, each device is given an energy budget and a number of bits, and has to choose its own transmit power in order to optimize a local utility increasing in remaining energy and decreasing in transmission time. Since the uplink channels are governed by a Markovian process, the design of the optimal protocol naturally leads to a stochastic differential game of large dimensions, a notoriously difficult problem to solve. We propose a novel modeling via the Mean-Field Game approximation, where the number of users is taken to infinity and their actions remain coupled only via average entities, allowing us to recover a set of equations which can be solved numerically to provide the optimal protocol.

II. RELATED WORK

The problem of distributed power control in the uplink of a single cell using a CDMA MAC has been modeled as a stochastic differential game in [5]. The players in this game are the transmitters who adapt their power level to the quality of their time-varying link with the receiver, their battery level, and the strategy updates of the other transmitters. The paper is of theoretical nature providing a simple sufficient condition for the existence of a Nash equilibrium in this game. As the uniqueness and determination of equilibria are difficult issues in general, especially when the number of players goes large, the paper addresses two special cases: the single player case and the large number of players case. The latter case is treated with a MFG approach for which reasonable sufficient conditions for convergence and uniqueness are provided. Illustrative numerical results which indicate that this MFG approach can lead to significant gains in terms of energy efficiency are shown in [6].

In this paper, we follow a similar model as in [5] and [6]. However, we shift our attention to Quality of Service considerations and more specifically the need to transmit a given number of information bits with a small delay. Latency requirements are important in a wide range of applications and receive considerable attention in 5G networks design. The problem of meeting latency requirements over wireless links is particularly challenging due to channel quality fluctuations which require adjustments to the transmission parameters (transmit power, modulation and coding scheme, etc) to achieve an optimum trade-off between transmission rate, packet error rate and energy efficiency [7] [8].

A recent paper [9] applies MFG techniques to develop a transmit power selection policy with the goal to optimize a combined cost of average queue length and energy consumption. Although the problem is formulated in the general case, in order to keep the solution mathematically tractable, a bufferless queue and a two-state channel are considered. In our paper we tackle the bufferless problem and consider a more general channel model which is similar to the channel model adopted in [5] [6]. We keep the mathematical solution tractable by applying an appropriate optimization objective which leads to a stationary solution, thus removing the dependence on time.

III. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a large number K of wireless devices which need to transmit information to the same base station receiver using a NOMA scheme. Each device receives at random times a number of bits that it needs to transmit to the common receiver and a certain energy budget for transmitting those bits. To make the model as generic as possible, the number of bits and energy budget are randomly drawn from a known probability distribution on $[0, B_{max}] \times [0, E_{max}]$. At any time t the i -th device can choose its own transmit power $p_i(t) \in [0, P_{max}]$. The achievable instantaneous transmission rate is given by

$$r_i(t) = Rf(\gamma_i(p_1(t), p_2(t), \dots, p_K(t))) \quad (1)$$

where R is a constant (in bits/s) and γ_i is the Signal to Interference plus Noise Ratio (SINR) after Matched-Filter detection at the receiver of a random CDMA scheme [10, Sec. 12.2] expressed as

$$\gamma_i(p_1(t), p_2(t), \dots, p_K(t)) = \frac{p_i(t)H_i(t)}{\sigma^2 + \frac{1}{M} \sum_{j \in \mathcal{K}, j \neq i} p_j(t)H_j(t)} \quad (2)$$

where $H_i(t)$ is a parameter representing the quality of the channel between the i -th transmitter and the receiver, σ^2 is a constant which models the communication noise effects at the receiver, M is the length of each CDMA spreading code (with the common assumption that as K grows large, $\frac{K}{M} \rightarrow c$, with c being a finite constant [10, Sec. 12.2]), and $f(\cdot)$ is a function which depends on the transmitter's capabilities and knowledge of the current SINR. Sophisticated transmitters which can use many modulation and coding schemes and adapt the used scheme to the current SINR can approximate the Shannon capacity limit. In this case $f(x) = \log_2(1+x)$. For simpler transmitters employing a single modulation and coding scheme, $f: \mathbb{R}^+ \rightarrow [0, 1]$ is a sigmoidal function which represents the bit success rate. In general, other choices of smooth increasing functions can be used.

The channel quality $H_i(t)$ evolves according to a random process which depends on many factors. A vast literature on modeling the evolution of the channel as a random process exists. We are going to adopt a commonly used model which is both representative of practical scenarios and amenable to mathematical analysis: different channel qualities are assumed to be independent and identically distributed (i.i.d) and each $H_i(t)$ is assumed to follow a reflected Brownian motion in the interval $[H_{min}, H_{max}]$. Note however, that our problem formulation and solution does not depend on the nature of the random process $H_i(t)$ and can be easily applied to a vast range of different random processes.

The dynamics of the system can be modeled as follows:

$$\begin{aligned} dE_i(t) &= -p_i(t)dt \\ dB_i(t) &= -r_i(t)dt \\ dH_i(t) &= \sqrt{2\nu}d\mathbb{W}_i(t) \end{aligned} \quad (3)$$

where $E_i(t)$ is the amount of remaining energy budget for device i and $B_i(t)$ is the amount of information (in number of bits¹) that device i has still to transmit, both at time t . $\mathbb{W}_i(t)$ denotes a Wiener process on a given probability space modeling the fluctuations of the channel quality H_i (fading) and the parameter $\nu > 0$ describes the intensity of the fading (it can be used to capture slow or fast fading). Note that in the above dynamics, the actions of each device affect the states of all other devices through the dependence of $r_i(t)$ to all the $p_i(t)$, $i = 1, \dots, K$.

The goal of each device is to transmit all its information bits within its energy budget and with an optimal trade-off between energy consumption and the time needed to

¹We assume a fluid model under which the number of bits is seen as a continuous variable

complete the transmission. We can model this phenomenon as a Stochastic Differential Game (SDG), that is, K coupled stochastic optimal control problems. We assume that device i will exit the game once it has transmitted all the information, i.e., when $B_i(t) = 0$ or when it is out of energy, i.e., $E_i(t) = 0$ (whichever comes first). We denote by τ_i the first time at which one of these conditions is satisfied. Each device faces the stochastic optimal control problem:

$$\sup_{0 \leq p_i(0 \rightarrow \tau_i) \leq P_{max}} \mathbb{E} \left[\int_0^{\tau_i} -\theta e^{-\lambda t} dt + e^{-\lambda \tau_i} \psi(E_i(\tau_i), B_i(\tau_i)) \right] \quad (4)$$

where $\psi(\cdot)$ is an appropriate terminal utility which is an increasing function of the amount of remaining energy and a decreasing function in the number of information bits not transmitted at the exit time, e.g., $\psi(E_i(\tau_i), B_i(\tau_i)) = E_i(\tau_i) - B_i(\tau_i)$, $\theta > 0$ is a parameter of the model that puts appropriate weights on the utility depending on the exit time (the first term) and depending on the exit cost (the second term), and λ is the inter-temporal preference rate of the devices (i.e., the weight they put on the present versus the future). The intuition behind the utility function in (4) is that it penalizes devices the longer they stay in the system (through the first term), and the more energy they have consumed and the less bits they have transmitted when they exit the system (through the second term). The multiplicative factor $e^{-\lambda t}$ guarantees that the average cost or utility function stays finite. Let us note that τ_i is not a control but it is a function of the whole trajectory of device i , thus a function of p_i and of $(p_j)_{j \neq i}$ (because of the coupling).

In this infinite horizon SDG, the control of a device i is its transmit power $p_i(t)$ which is allowed to depend not only on time, but on its own state (E_i, B_i, H_i) and on the states of all other devices in the system. This is an extremely complicated problem not only because of the huge number of variables and couplings involved but also because in practical implementations the communication overhead of relaying all the state of each device to each other device will be prohibitively large. In order to arrive at a practical and efficient scheme, we propose below to use a MFG limit for this game.

In practical situations, the same devices can enter and exit the game over and over again as their transmitters get some data to transmit at random time instances from the applications running on the device. As we want to keep the model simple, without considering the need for buffering data received in more than one round, we assume that devices that are in the game (having still data to transmit) cannot receive new data from their application layer. We assume that such a data-blocking event happens with an extremely small probability as the mean exit time (during which the device is in a blocking-additional-data mode) is much smaller than the minimum inter-arrival time of new data.

IV. MEAN FIELD GAME ANALYSIS

It is well known [5] [6] that this type of SDG can be simplified by considering the MFG regime as $K \rightarrow \infty$. In

this setting, a player has interactions with the other players only through mean field terms (averaged quantities), which in our problem is the SINR experienced by the player. Because the coupling between the players only appears through an averaged quantity, the optimization problem faced by a generic player only depends on the distribution m of other players.

Moreover, let us note that in the SDG with K players, as devices reach their exit time, they exit the game. Thus the number of players in the game is decreasing over time, and will eventually reach 0, making the game trivial. We assume in this MFG that new players arrive randomly in the game according to a Poisson process and that the state of a newly arriving player is randomly drawn from a distribution $m_s(E, B, H)$. Because the problem has an infinite horizon and the distribution of arriving player m_s as well as the terminal utility $\psi(E, B)$ do not depend on time, the Nash equilibrium is stationary, i.e., it does not depend on time. If we denote by u the value function of the MFG (and by m the non-normalized density of the devices), then a stationary Nash equilibrium is described by a solution of the following stationary system of PDE (the first PDE is the infinitesimal dynamic programming principle and the second one the infinitesimal local conservation of the number of devices) on $\Omega = (0, E_{max}) \times (0, B_{max}) \times (H_{min}, H_{max})$:

$$\lambda u - \nu \partial_{HH} u + p^* \partial_E u + R f\left(\frac{Hp^*}{\Gamma}\right) \partial_B u + \theta = 0 \quad (5)$$

$$-\nu \partial_{HH} m - \partial_E (p^* m) - R \partial_B \left(f\left(\frac{Hp^*}{\Gamma}\right) m \right) = m_s \quad (6)$$

where, p^* is the optimal strategy, i.e., the solution of

$$p^*(E, B, H) = \operatorname{argmax}_{0 \leq p \leq P_{max}} \left\{ -p \partial_E u - R f\left(\frac{Hp}{\Gamma}\right) \partial_B u \right\} \quad (7)$$

and Γ is representing the mean field coupling of the two PDE through the interference experienced by a generic player which is given by:

$$\Gamma = \sigma^2 + c \int \int \int_{\Omega} h p^*(E, B, H) m(E, B, H) dB dE dH \quad (8)$$

The above system of PDE is subject to the following boundary conditions:

$$\begin{aligned} u(0, B, H) &= -B \\ u(E, 0, H) &= E \\ \partial_H u(E, B, H_{min}) &= 0 \\ \partial_H u(E, B, H_{max}) &= 0 \end{aligned} \quad (9)$$

Most of the boundary conditions above are self-evident. The last two boundary conditions are a direct consequence of the fact that the channel state H is modeled as a reflected Brownian motion in $[H_{min}, H_{max}]$.

The above system of PDE ((5) - (9)) is typical in MFG. We refer the reader to [2] [3] for more details on this system. Equation (5) is known as the Hamilton-Jacobi-Bellman (HJB) equation and given a fixed mean field Γ , its solution is the

value function of a generic player facing (4) whose dynamics, under a choice of control p , evolve according to:

$$\begin{aligned} dE(t) &= -pdt \\ dB(t) &= -Rf\left(\frac{pH}{\Gamma}\right)dt \\ dH(t) &= \sqrt{2\nu}d\mathbb{W}(t) \end{aligned}$$

where $(\mathbb{W}(t))_{t \geq 0}$ is a reflected brownian motion in $[H_{min}, H_{max}]$. It is classical to check by a verification argument that such a trajectory is optimal for the players. The Fokker-Planck (FP) equation (6) is solved by the non-normalized density of devices, provided that with energy E , data left to transmit B and quality of channel H , all the devices use the control $p^*(E, B, H)$. Moreover let us note that we can solve the problem of a given device by solving this system. Indeed if a device is in the state (E, B, H) it uses the optimal control $p^*(E, B, H)$. Thus, the state of a device being at (E_0, B_0, H_0) at time t_0 evolves according to:

$$\begin{aligned} dE(t) &= -p^*(E(t), B(t), H(t))dt; \\ dB(t) &= -Rf\left(h\frac{p^*(E(t), B(t), H(t))}{\Gamma}\right)dt; \\ dH(t) &= \sqrt{2\nu}d\mathbb{W}(t); \\ E(t_0) &= E_0; B(t_0) = B_0; H(t_0) = H_0; \end{aligned}$$

while $\min(E(t), B(t)) > 0$. Hence, knowing the solution of the MFG system, the players can compute their optimal control depending only on their own state, provided that all the other devices use the control $p^*(E, B, H)$ while in the state (E, B, H) , making their anticipation of the mean field (the value of Γ) correct. We recall that we are looking for Nash equilibria of the game.

This system of equations has no closed form solution and must be solved numerically. We provide below some explanations on how to solve the MFG system numerically, see [11] for more information. In order to solve the PDE problem at hand we propose the following iterative method:

We start from a given value of Γ and we fix a parameter $1 \geq \eta > 0$. We then proceed as follows:

- Solve the two PDE (5) and (6) using a finite differences scheme. Note that for a fixed Γ , (5) does not depend on the solution of (6), thus we solve first (5) and then (6).
- Calculate Γ' using equation (8), and update Γ according to:

$$\Gamma = (1 - \eta)\Gamma + \eta\Gamma' \quad (10)$$

- Iterate until $|\Gamma - \Gamma'| < \epsilon$, where ϵ is a very small positive number setting the convergence criterion.

Although we do not have any formal proof of convergence, the proposed iterative method seems to always converge in the simulation experiments we have run.

To solve the system (5), (6), (7) we work on a uniform grid G_l of l^3 points on $[0, E_{max}] \times [0, B_{max}] \times [H_{min}, H_{max}]$. A function on G_l is associated to an element of $(\mathbb{R}^N)^3$, where $N = \frac{1}{l}$. The steps on the grid are of size $dE = \frac{E_{max}}{N}$; $dB =$

$\frac{B_{max}}{N}$; $dH = \frac{H_{max} - H_{min}}{N}$. First, we find a zero of the function $F(u)$ defined on $(\mathbb{R}^N)^3$ by:

$$\begin{aligned} F(u)_{e,b,h} &= \lambda u_{e,b,h} + \theta + p_{e,b,h}^* \frac{u_{e,b,h} - u_{e-1,b,h}}{dE} + \\ &Rf\left(\frac{H_h p_{e,b,h}^*}{\Gamma}\right) \frac{u_{e,b,h} - u_{e,b-1,h}}{dB} - \\ &\nu \frac{u_{e,b,h+1} - 2u_{e,b,h} + u_{e,b,h-1}}{dH^2} \end{aligned} \quad (11)$$

where $1 \leq e, b, h \leq N$; $p_{e,b,h}^*$ is given by:

$$p_{e,b,h}^* = \operatorname{argmax}_{0 \leq p \leq P_{max}} \left\{ -p \frac{u_{e,b,h} - u_{e-1,b,h}}{dE} - Rf\left(\frac{H_h p}{\Gamma}\right) \frac{u_{e,b,h} - u_{e,b-1,h}}{dB} \right\} \quad (12)$$

and H_h is given by:

$$H_h = H_{min} + \frac{H_{max} - H_{min}}{N} h.$$

We use as boundary conditions the standard conventions that for any $1 \leq e, b, h \leq N$:

$$\begin{aligned} u_{0,b,h} &= -B_{max} \frac{b}{N} \\ u_{e,0,h} &= E_{max} \frac{e}{N} \\ u_{e,b,0} &= u_{e,b,2} \\ u_{e,b,N+1} &= u_{e,b,N-1} \end{aligned}$$

We then use a Newton algorithm to find a solution of $F(u) = 0$. The function F is the discretization of the HJB equation, hence solving $F(u) = 0$ yields a numerical solution of the HJB equation.

In order to solve the Fokker-Planck equation (6), we use the newly found solution of the HJB equation $u \in (\mathbb{R}^N)^3$, such that $F(u) = 0$. Then we define \tilde{F} by $\tilde{F}(u)_{e,b,h} = F(u)_{e,b,h} - \lambda u_{e,b,h} - \theta$, and we look at the linear problem:

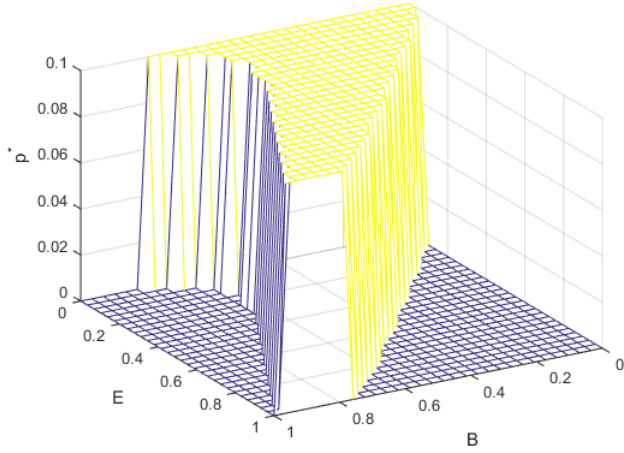
$$(D\tilde{F}(u))^T m = m_s^h$$

where $D\tilde{F}(u)$ stands for the differential of \tilde{F} at u , the unknown is $m \in (\mathbb{R}^N)^3$ and $m_s^h \in (\mathbb{R}^N)^3$ is the numerical approximation of m_s . This linear problem is the direct analogous of the FP equation.

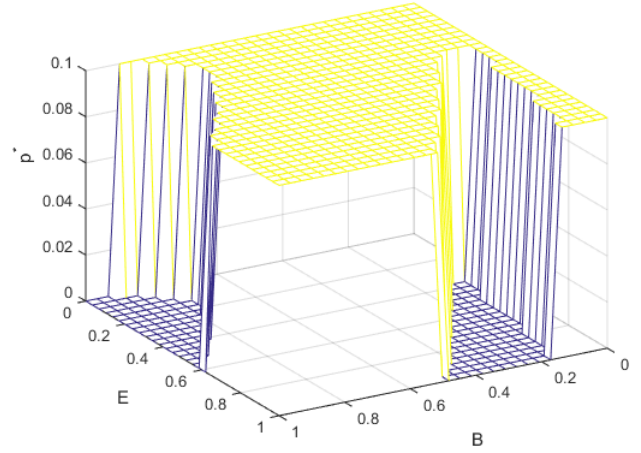
V. NUMERICAL RESULTS

In this section we provide numerical results that illustrate the control strategies obtained as a solution to our MFG formulation of the optimal transmit power problem. In order to obtain these results we have used the following parameters:

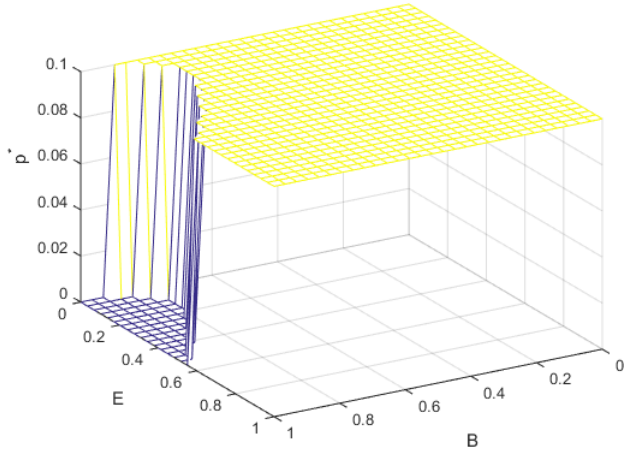
- $R = 8$ Mbps ($= 8 \times 10^6$ bps)
- $P_{max} = 0.1$ Watt
- $\sigma^2 = 1$
- $c = 500$
- $\nu = 0.28$
- $E_{max} = 1$ Joule
- $B_{max} = 1$ Kbit ($= 1000$ bits)



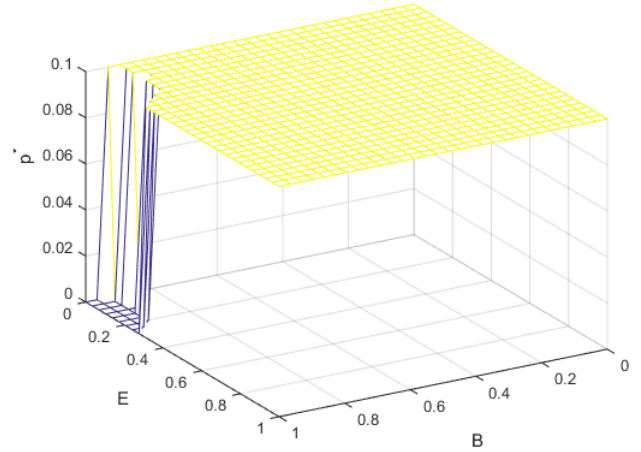
(a) $h = 1(H = 0.2)$



(b) $h = 5(H = 0.5862)$.



(c) $h = 6(H = 0.6828)$.



(d) $h = 11(H = 1.1655)$.

Fig. 1. Optimal transmit power as a function of remaining energy E and remaining bits to transmit B .

- $H_{min} = 0.2$
- $H_{max} = 3.0$
- $\theta = 0.101$
- $\lambda = 0.1$

We have also assumed a uniform source rate $m_s(E, B, H) = 0.1$ everywhere, a simple function $\psi(E_i(\tau_i), B_i(\tau_i)) = E_i(\tau_i) - B_i(\tau_i)$ and the Shannon capacity approximation for the achievable rate, i.e., that $f(x) = \log_2(1 + x)$.

Using the above parameters we have solved numerically the system of PDE characterizing the MFG as described in the previous section for $N = 30$. In Figure 1, we plot the optimal transmit power as a function of E and B for different values of h . Let us note that because we choose the parameters such that $\lambda^{-1}\theta > B_{max}$, there is no device which has an interest not to exit the game. Indeed, $-\lambda^{-1}\theta$ is the cost to stay in the game forever and $-B_{max}$ is the worst possible terminal cost.

A first important observation is that the optimal policy takes

the values 0 or P_{max} at almost all grid points. We can see that as the quality of the channel grows, the number of points in the (E, B) plane for which $p^*(E, B, H) = P_{max}$ grows as well (we don't plot p^* for high values of H , as for $h \geq 21$ it is $p^*(E, B, H) = P_{max}$ for all (E, B)). When a device experiences very low channel quality (Figure 1(a)), it doesn't want to spend energy to get some bits transmitted in two cases: (i) when it has a lot of energy left and few bits to transmit and (ii) when it has little to medium energy left and many bits to transmit. In the first case, the device has the luxury to wait for better channel conditions as it only has a few bits to transmit and hence it can complete their transmission in a small amount of time. In the latter case, the device has little energy left and prefers to wait for safer channel conditions. As the quality of the channel improves, both the areas corresponding to cases (i) and (ii) above shrink in size with the area corresponding to case (i) shrinking and disappearing faster (Figures 1(b), 1(c),

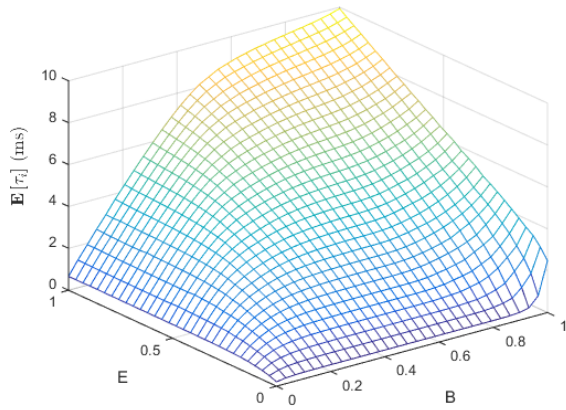


Fig. 2. Expected exit time (in milliseconds) as a function of energy budget E and received bits to transmit B . This is identical to the expected remaining time in the game as a function of remaining energy and remaining bits to transmit.

and 1(d)).

In Figure 2, we plot the expected exit time $\mathbb{E}[\tau_i(E, B)]$ averaged over channel quality, for a device that has received B bits to transmit with an energy budget of E Joules. Note that this is also the expected remaining time in the game for a device that at some point in time has B remaining bits to transmit and E Joules of remaining energy. As expected, $\mathbb{E}[\tau_i(E, B)]$ is an increasing function in both E and B .

VI. CONCLUSIONS AND DIRECTIONS FOR FUTURE WORK

The problem of minimizing energy usage while achieving QoS requirements in uncoordinated uplink transmissions for MTC users is a very complex stochastic control problem with multiple agents. We have proposed a MFG model for this type of problems that reduces the complexity by a great deal and is thus amenable to numerical solution. Our model is general enough to include generic rate functions, arbitrary energy and QoS requirements per user, different channel fading models, and design knobs for determining the importance of different performance goals.

A lot of extensions to the model presented in this paper seem possible and easy to compute. We are particularly interested to the case of non-identical and correlated channels which is both encountered in practice and of theoretical interest in MFG. Indeed, in MFG literature the correlated channels case is known as MFG with common noise and its solution involves the so-called master equation. This will be the subject of a future work. Other potential extensions include the case where the QoS quantity of interest is not the mean transmission delay but the probability that a given delay threshold is exceeded. Another interesting case is when arriving data from the application layer are stored in a buffer forming a FIFO queue and the objective is to control the queue length. Finally, we believe practical questions can be solved by applying MFG inspired policies in actual systems. Such practical questions include the

robustness of the control policies to various errors, including errors in estimating the channel quality at the transmitter and MFG approximation errors, i.e., how good is the control policy derived for the MFG limit ($K \rightarrow \infty$) when applied by a finite (yet large) number of transmitters.

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