# Beamforming capacity optimization for MISO systems with both Mean and Covariance Feedback

Efstathios Vagenas, Georgios S. Paschos and Stavros A. Kotsopoulos

***Abstract*—The beamforming capacity optimization problem in MISO systems, when the transmitter has both mean and covariance feedback of the channel, has been tackled only with the *SNR* maximization approach, which is known to give a sub-optimal solution. Numerical solutions of the full rank input covariance matrix, presented in the literature, are capable of tracking the beamforming vector only if it is the optimal capacity achieving solution. In this paper, we solve the beamforming capacity optimization problem by following an analytical approach that projects the beamforming vector on an orthonormal basis defined by the eigenvectors of the channel covariance matrix. The proposed formulation reduces the complexity of calculating the solution and provides intuition into the problem itself. In particular, we express the necessary conditions for beamforming capacity maximization as a system of two equations, which can be solved numerically very efficiently using the secant method. Surprisingly, our indicative numerical results for the $2 \times 1$ and $10 \times 1$ MISO systems, showed that for some cases the performance gain through beamforming capacity optimization compared to the *SNR* maximization approach can reach $0.4bps/$*Hz*. This means that the *SNR* maximization solution deviates considerably from the optimal beamforming vector. Finally, the optimality of the *SNR* maximization solution is also examined.**

***Index Terms*—Beamforming, Capacity, Input optimization, Rician fading channels.**

## I. Introduction

Systems with multiple antennas are known to significantly increase channel capacity, and thus they represent one of the most promising solutions for fast and reliable wireless communications [1]. In Multiple-Input Multiple-Output (MIMO) systems, channel knowledge is substantial for high capacity [2], [3]. Although the receiver may have perfect channel state information, realizing such knowledge at the transmitter is, in most instances of wireless communications, difficult or even impossible.

Instead of tracking the instantaneous state of the fading channel, it is more practical for the transmitter to rely on the statistical knowledge of the channel, [4]. As it was shown in [5], the optimal input is Gaussian distributed with zero-mean, while finding its optimal input covariance matrix leads to the transmitter optimization problem. The optimum covariance matrix is, in general, a full rank matrix whose eigenvectors and eigenvalues have to be optimized simultaneously [6]–[9]. Eigenvectors and eigenvalues can only be optimized separately in the special cases of either mean or covariance channel knowledge. Such cases were studied in [10]–[17]. However, as stated in [18], if the channel has to be described by both mean and covariance, no closed form solution for capacity maximization exists even for MISO systems.

Limiting the rank of the input covariance matrix to unity, leads to the widely known beamforming scenario. Practically, beamforming is desirable because it reduces system complexity significantly (i.e. low coding effort-cheap transmitters). Beamforming/precoding reduces a MIMO system into a Single-Input Single-Output (SISO) system, which means that a scalar codec technology can be used. Moreover, this technique gives close to optimal results either at low Signal-to-Noise Ratio ($SNR$), or at relatively small angular spreads of the channel (i.e. narrow spatial spread), a situation that typically occurs in outdoor channels, [19]. The necessary conditions for the optimality of beamforming were presented in [12], [20], [21]. The beamforming capacity optimization problem was solved in [19], [22] for MISO systems with either channel mean information or channel covariance information at the transmitter and was further studied in [23] in the context of wideband OFDM. A generalization of that solution for MIMO systems was presented in [15]. An interesting study of the low-SNR scenario can be found in [24] where a closed-form expression for the optimal beamforming direction was presented. However, finding the optimal beamforming vector in channels with both mean and covariance feedback remains an open problem even for MISO systems.

In this paper, we consider a MISO system which uses beamforming as its transmit strategy, as in [15], [23], and we give a solution for the optimal beamforming vector when both mean and covariance feedback are available at the transmitter and when both are necessary to describe the channel (Rician fading channel). Furthermore, the receiver is assumed to have perfect channel knowledge. The results of [19], [22], [23], with only mean or covariance knowledge, are special cases of the beamforming maximization problem and thus are included in our solution. Although numerical solutions of the full rank input covariance matrix already exist in literature, they can not be applied to beamforming optimization because these algorithms track the beamforming vector only if it is optimal for capacity maximization. *SNR* maximization is the only known sub-optimal solution, [23]. We tackle this problem by approaching its solution analytically and projecting the optimal beamforming vector on an orthonormal basis defined by the eigenvectors of the channel covariance matrix. Following this procedure, first we alleviate the complexity of the beamforming vector (from the $N$ complex optimization variables to $N$ real valued variables) and second, we proceed by reducing the problem from a system of $N$ equations to a system of 2 equations with 2 variables. The rest $N-2$

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variables are calculated by simple substitutions. Although this system cannot be solved analytically in closed form, it can be solved efficiently, using the numerical secant method, making the problem solvable even in real time by devices with limited processing power. Our results are extended to the special cases of low and high power, where the equations are further simplified. This method allows us to examine how well the *SNR* maximization solution performs in terms of capacity and to quantify the tradeoff between the *SNR* maximization and capacity maximization solutions. Besides its practical importance, beamforming capacity optimization is of great theoretical importance since its solution can potentially point toward the general solution of the full rank capacity achieving input covariance matrix in Rician fading channels.

The paper is organized as follows. In Section II, the problem is formulated and the beamforming vector is analyzed in an orthonormal basis. In Section III the necessary conditions for optimality are presented. In Section IV the necessary conditions are transformed to a system of two equations. Section V presents numerical results and Section VI concludes the paper.

## II. System model

Consider an $N\times 1$ MISO system operating over a frequency non-selective fading channel. The complex baseband representation of the received signal is given by

$$y=\frac{\sqrt{P}}{N}\mathbf{h}\mathbf{x}+n,$$

where $\mathbf{x}$ is the $N\times 1$ input vector, $\mathbf{h}$ is the $1\times N$ channel vector, $n$ is the additive white zero-mean complex Gaussian noise with power $\sigma_n^2$ and $P$ is the total power of a transmitted signal. In order to achieve capacity, we feed the system with a complex Gaussian random vector (as in [5]) with covariance matrix

$$\mathbf{Q}=\mathbb{E}\left[\mathbf{x}\mathbf{x}^H\right],$$

where $\mathbb{E}[.]$ denotes the expectation and $.^H$ denotes the Hermitian transpose, or the conjugate transpose of the matrix. Next, assuming that $\mathbf{h}$ is perfectly known to the receiver, the obtained ergodic capacity per bandwidth (in bits/s/Hz) is

$$C=\mathbb{E}_{\mathbf{h}}\left[\log_2\left(1+\frac{P}{\sigma_n^2}\mathbf{h}\mathbf{Q}\mathbf{h}^H\right)\right],$$

where the expectation is taken over the channel vector $\mathbf{h}$. The covariance matrix $\mathbf{Q}$ can be decomposed with the help of eigenanalysis (Schur decomposition) to

$$\mathbf{Q}=\mathbf{U}_Q\mathbf{\Lambda}_Q\mathbf{U}_Q^H,$$

where $\mathbf{U}_Q$ is a square matrix of the eigenvectors of $\mathbf{Q}$ and $\mathbf{\Lambda}_Q$ is a diagonal matrix of the eigenvalues of $\mathbf{Q}$. If the system chooses beamforming as the transmit strategy, then $\mathbf{Q}=\mathbf{v}^H\mathbf{v}$, where $\mathbf{v}\in\mathbb{C}^N$ is the $1\times N$ transmit beamforming vector, with $\|\mathbf{v}\|=1$. In this case, the ergodic beamforming capacity has been derived in [23]

$$\begin{aligned}C_{bf}&=\mathbb{E}_{\mathbf{h}}\left[\log_2\left(1+\frac{P}{\sigma_n^2}\mathbf{h}\mathbf{v}^H\mathbf{v}\mathbf{h}^H\right)\right]\\&=\mathbb{E}_{\mathbf{h}}\left[\log_2\left(1+\frac{P}{\sigma_n^2}\left|\mathbf{h}\mathbf{v}^H\right|^2\right)\right].\end{aligned}\tag{1}$$

In the above $|.|$ denotes the modulus and $\|.\|$ denotes the length (or square norm) of the corresponding vector. If each element of the channel $\mathbf{h}$ is described by a complex Gaussian random variable, then $\mathbf{h}=\boldsymbol{\mu}+\mathbf{h}_w\mathbf{K}^{\frac{1}{2}}$ where $\boldsymbol{\mu}\in\mathbb{C}^N$ is the $1\times N$ channel mean vector, $\mathbf{h}_w$ is a zero-mean spatially-white (ZMSW) channel and $\mathbf{K}$ is an Hermitian positive definite $N\times N$ covariance matrix. Thus $z(\mathbf{v})\doteq\mathbf{h}\mathbf{v}^H=\boldsymbol{\mu}\mathbf{v}^H+\mathbf{h}_w\mathbf{K}^{\frac{1}{2}}\mathbf{v}^H$ is also a complex Gaussian random variable and hence the modulus $|z(\mathbf{v})|$ is Rice distributed with the following probability density function

$$p_{|z(\mathbf{v})|}(x)=\frac{x}{\sigma_z^2(\mathbf{v})}I_0\left(\frac{m_z(\mathbf{v})}{\sigma_z^2(\mathbf{v})}x\right)e^{-\frac{x^2+m_z^2(\mathbf{v})}{2\sigma_z^2(\mathbf{v})}},x\geq 0,\tag{2}$$

where $I_0(.)$ is the modified Bessel function of the first kind with zero order and

$$m_z(\mathbf{v})=\left|\boldsymbol{\mu}\mathbf{v}^H\right|\tag{3}$$

$$\sigma_z^2(\mathbf{v})=\frac{1}{2}\mathbf{v}\mathbf{K}\mathbf{v}^H,\tag{4}$$

as in [23], both positive real numbers. Note here that the equations (3) and (4) are not bijections, i.e. we can find $\mathbf{v}_1\neq\mathbf{v}_2$ such that $m_z(\mathbf{v}_1)=m_z(\mathbf{v}_2)$ and $\sigma_z^2(\mathbf{v}_1)=\sigma_z^2(\mathbf{v}_2)$. Also, note that $m_z$ and $\sigma_z^2$ correspond to the mean and variance of the random variable $|z|$ and that for presentation reasons we have adopted a different notation instead of $m_{|z|}$ and $\sigma_{|z|}^2$.

Thus, for an $N\times 1$ MISO system which applies transmit beamforming strategy and has partial channel knowledge (i.e. the transmitter knows the statistics of the channel $\boldsymbol{\mu}$, $\mathbf{K}$ and the receiver knows the channel exactly and instantaneously), the capacity optimization problem can be formulated as follows:

<u>Beamforming Capacity maximization:</u>

$$\begin{aligned}C_{bf}=&\underset{\mathbf{v}}{\text{maximize}}\,\mathbb{E}_z\left[\log_2\left(1+\frac{P}{\sigma_n^2}\left|z(\mathbf{v})\right|^2\right)\right]\\&\text{subject to}\quad\|\mathbf{v}\|=1.\end{aligned}\tag{5}$$

where the distribution of $|z(\mathbf{v})|$ is given by (2). The capacity function (5) is an increasing function of all the parameters $P, m_z(\mathbf{v})$ and $\sigma_z^2(\mathbf{v})$ and strictly concave with respect to $\mathbf{v}$, [21]. Hereinafter, we will assume $\sigma_n^2=1$ for simplicity. Thus, $P$ corresponds to the *SNR*.

The problem (5) above is fairly complex for real time calculations (see [23] for detailed information) since solving the integral and using numerical optimization methods in a wireless transmitter is not feasible. In the following section we give a semi-analytic solution that extremely simplifies the problem and provides insight about the performance of the suboptimal solutions with respect to the optimal for several cases.

### A. Changing the optimization variables

We relate the statistical parameters of the arising Rician channel $m_z(\mathbf{v}),\sigma_z^2(\mathbf{v})$ with the generalized angles that correspond to the vectors $\mathbf{v}$, $\boldsymbol{\mu}$ and the eigenvectors of the channel covariance matrix $\mathbf{K}$. Since the covariance matrix of the channel is an Hermitian positive definite matrix, it can be decomposed to

$$\mathbf{K}=\mathbf{U}_K\mathbf{\Lambda}_K\mathbf{U}_K^H,\tag{6}$$

where the column vectors of $\mathbf{U}_K$, $\mathbf{u}_K^i \in \mathbb{C}^N,\ i = 1, \ldots, N$ are the eigenvectors of $\mathbf{K}$ and $\mathbf{\Lambda}_k$ is a diagonal matrix consisting of the eigenvalues $\lambda_i \geq 0$ of $\mathbf{K}$. Note that, since $\mathbf{K}$ is Hermitian, the eigenvectors (columns in $\mathbf{U}_K$ or rows in $\mathbf{U}_K^H$) form an orthonormal basis in $\mathbb{C}^N$. Also, note that $\mathbf{U}_K\mathbf{U}_K^H = \mathbf{I}_N$, where $\mathbf{I}_N$ is the $N \times N$ identity matrix.

Using the orthonormal basis of the row vectors of $\mathbf{U}_K^H$, we can express the vectors $\boldsymbol{\mu}$, $\mathbf{v}$ as

$$\begin{aligned}\mathbf{v} &= \cos(\phi_1)e^{j\omega_1^v}{\mathbf{u}_K^1}^H + \cdots + \cos(\phi_N)e^{j\omega_N^v}{\mathbf{u}_K^N}^H \\ \boldsymbol{\mu} &= \|\boldsymbol{\mu}\|\cos(\theta_1)e^{j\omega_1^\mu}{\mathbf{u}_K^1}^H + \cdots + \|\boldsymbol{\mu}\|\cos(\theta_N)e^{j\omega_N^\mu}{\mathbf{u}_K^N}^H,\end{aligned} \tag{7}$$

where $\phi_i$, $\theta_i$ are the generalized angles of vectors $\mathbf{v}$, $\boldsymbol{\mu}$ with vector ${\mathbf{u}_K^i}^H$ (direction cosine analysis), and $\omega_i^v$, $\omega_i^\mu$ are the complex arguments of $\mathbf{v}\mathbf{u}_K^i$ and $\boldsymbol{\mu}\mathbf{u}_K^i$ respectively. The generalized angle between any two vectors $\mathbf{v}_1, \mathbf{v}_2 \in \mathbb{C}^N$ is defined as $\cos(\phi) = \frac{|\mathbf{v}_1\mathbf{v}_2^H|}{\|\mathbf{v}_1\|\|\mathbf{v}_2\|}, 0 \leq \phi \leq \frac{\pi}{2}$. For the generalized angles $\phi_i$, that characterize the projection of a vector onto an orthonormal basis, it holds that

$$\sum_{i=1}^{N} \cos^2\phi_i = 1. \tag{8}$$

Equation (8) is also equivalent with the constraint of (5). From (4), (6) and (7) we obtain

$$\begin{aligned}\sigma_z^2(\mathbf{v}) &= \frac{1}{2}\mathbf{v}\{\mathbf{u}_K^1, \ldots, \mathbf{u}_K^N\}\mathbf{\Lambda}_K\{{\mathbf{u}_K^1}^H, \ldots, {\mathbf{u}_K^N}^H\}^T\mathbf{v}^H \\ &= \frac{1}{2}\sum_{i=1}^{N}\lambda_i\mathbf{v}\mathbf{u}_K^i{\mathbf{u}_K^i}^H\mathbf{v}^H = \frac{1}{2}\sum_{i=1}^{N}\lambda_i\left|\mathbf{v}\mathbf{u}_K^i\right|^2 \\ &= \frac{1}{2}\sum_{i=1}^{N}\lambda_i\cos^2\phi_i\end{aligned} \tag{9}$$

Also from (3) and (7) and taking into account that $\mathbf{u}_K^i{\mathbf{u}_K^i}^H = 1$ and $\mathbf{u}_K^i{\mathbf{u}_K^j}^H = 0, \forall i \neq j$, we have

$$\begin{aligned}m_z(\mathbf{v}) &= \left|\boldsymbol{\mu}\mathbf{v}^H\right| \\ &= \|\boldsymbol{\mu}\|\left|\sum_{i=1}^{N}\cos\theta_i\cos\phi_i e^{j\left(\omega_i^\mu - \omega_i^v\right)}\right|.\end{aligned}$$

Observe in (9) that $\sigma_z^2$ does not depend on $\omega_i^v, i = 1, \ldots, N$. Thus, by fixing $\sigma_z^2$ and $\cos\phi_i, i = 1, \ldots, N$, in order to maximize the capacity, one has simply to maximize $m_z$ using $\omega_i^v$s, which evidently happens by selecting $\omega_i^v = \omega_i^\mu, \forall i = 1, \ldots, N$. In fact, it suffices to set $\omega_i^\mu - \omega_i^v = \xi, \forall i = 1, \ldots, N$ where $\xi$ is an arbitrary angle, in order to align all vectors (complex numbers) so that the length of the sum is maximized; here we have chosen $\xi = 0$. Using this assignment, we finally get

$$m_z = \|\boldsymbol{\mu}\|\sum_{i=1}^{N}\cos\theta_i\cos\phi_i. \tag{10}$$

Thus the maximization problem of (5) is transformed with the help of (9), (10) to an optimization problem with respect to the vector $\{\phi_1, \ldots, \phi_N\}$ and subject to the constraint of (8).

## III. Necessary conditions for Capacity Maximization

To approach the problem (5) we substitute equation (8) to (9), (10) and get the following unconstrained optimization problem

$$\text{maximize}\ \ \mathbb{E}_z\left[\log_2\left(1 + \frac{P}{\sigma_n^2}|z|^2\right)\right],$$

where the parameters of the Rice-distributed random variable $|z|$ are

$$\begin{aligned}2\sigma_z^2 &= \sum_{i=1}^{N-1}(\lambda_i - \lambda_N)\cos^2\phi_i + \lambda_N \\ m_z &= \|\mu\|\left(\sum_{i=1}^{N-1}\cos\theta_i\cos\phi_i + \cos\theta_N\sqrt{1 - \sum_{i=1}^{N-1}\cos^2\phi_i}\right).\end{aligned}$$

Let $\Gamma(.,.)$ be the incomplete Gamma function, we are ready to state the first result.

*Theorem 1 (Stationary Points):* The necessary (but not sufficient) condition for $C_{bf}$ given by (5) to have a local extremum is given by the following system of non-linear equations

$$m_z\frac{\frac{\partial m_z}{\partial\phi_i}}{\frac{\partial\sigma_z^2}{\partial\phi_i}} + 1 = \frac{m_z^2}{2\sigma_z^2}\left(1 - \zeta\left(m_z^2, \sigma_z^2, P\right)\right), i = 1, \ldots, N-1, \tag{11}$$

where

$$\zeta\left(m_z^2, \sigma_z^2, P\right) \doteq \frac{1}{2P\sigma_z^2}\frac{\sum_{k=0}^{\infty}\frac{\left(\frac{m_z}{2\sqrt{P}\sigma_z^2}\right)^{2k}}{k!}\Gamma\left(-k-2, \frac{1}{2P\sigma_z^2}\right)}{\sum_{k=0}^{\infty}\frac{\left(\frac{m_z}{2\sqrt{P}\sigma_z^2}\right)^{2k}}{k!}\Gamma\left(-k-1, \frac{1}{2P\sigma_z^2}\right)}. \tag{12}$$

*Proof:* See Appendix A. ■

The sums in (12) are positive and fast converging; ten terms are sufficient, giving a more than three digit accuracy.

Note that the solution of (11) depends on the available power $P$ except from the cases (a) $m_z = 0$ (zero-mean Gaussian channel) and (b) $\left(\frac{1}{\sigma^2}\right)' = 0$ (corresponding to uncorrelated channel $\mathbf{K} = \mathbf{I}$). This is in accordance with [14], [23] where these two specific cases are inspected.

*Theorem 2:* The capacity $C_{bf}$ of (5) has at least one stationary point and its maximum can only be attained at a stationary point (despite the fact that it is defined in a bounded domain).

*Proof:* See Appendix B. ■

Obviously according to Theorems 1, 2, a unique solution of (11) implies that the capacity of (5) will have a unique global maximum. If we have multiple solutions, one of these solutions will provide the maximum. In any case, the optimal beamforming vector is given from (7) with $\omega_i^v = \omega_i^\mu, \forall i = 1, \ldots, N$. $\phi_1, \ldots, \phi_{N-1}$ constitute the solution of the system of equations in (11) and $\phi_N$ is obtained using the constraint of (8).

Note that the optimal beamforming vector is not unique. If $\mathbf{v}^\star$ is optimal, then $\mathbf{v}^\star e^{\mathrm{i}a}$, where $a$ is any angle, is also optimal since the value of (9) and (10) does not change.

By looking into specific asymptotics for $P$, however, we can yield two important particular cases.

*Corollary 1 (low power regime):* Let $P \to 0$, then $\zeta \to 1$ and the condition in the theorem 1 can be rewritten as

$$m_z \frac{\partial m_z}{\partial \phi_i} + \frac{\partial \sigma_z^2}{\partial \phi_i} = 0, \qquad i = 1, \dots, N-1 \tag{13}$$

Note that the solution of Corollary 1 corresponds to the *SNR* maximization solution, presented in [23]. This is rational since $\log(x) \approx x$ for very small $x$. Thus, when $P \to 0$, capacity maximization of (5) coincides with *SNR* maximization. However, the resulting optimal beamforming vector is different from the one given in [23], but this is not surprising as we explained above.

*Corollary 2 (high power regime):* Let $P \to \infty$, then the condition of theorem 1 can be rewritten as

$$e^{\frac{m_z^2}{2\sigma_z^2}} = 1 - \frac{m_z}{2\sigma_z^2} \frac{\partial \sigma_z^2}{\partial \phi_i} \left( \frac{\partial m_z}{\partial \phi_i} \right)^{-1}, i = 1, \dots, N-1 \tag{14}$$

A proof of Corollary 2 is given in the Appendix C. Let $\underline{\boldsymbol{\phi}}$ and $\overline{\boldsymbol{\phi}}$ be vectors containing the roots of (13) and (14) respectively and $\boldsymbol{\phi}^\star$ the roots of (11). Our simulations indeed show that $\underline{\boldsymbol{\phi}} \leq \boldsymbol{\phi}^* \leq \overline{\boldsymbol{\phi}}$ (where $\leq$ denotes element-wise ordering), though a proof is not mathematically tractable.

## IV. System of two Equations leads to Optimal Solution

From (11), equating by parts after some mathematical manipulations we obtain the following very useful set of equations

$$\frac{\frac{\partial m_z}{\partial \phi_i}}{\frac{\partial \sigma_z^2}{\partial \phi_i}} = \frac{\frac{\partial m_z}{\partial \phi_j}}{\frac{\partial \sigma_z^2}{\partial \phi_j}}, i \neq j, i, j \in \{1, \dots, N-1\}.$$

Using the above, after executing the differentiations, we have

$$(\lambda_1 - \lambda_N) \frac{\cos\theta_i}{\cos\phi_i} = (\lambda_i - \lambda_N) \frac{\cos\theta_1}{\cos\phi_1} + (\lambda_1 - \lambda_i) \frac{\cos\theta_N}{\cos\phi_N}, \quad i = 2, \dots, N-1. \tag{15}$$

Note that all the $\cos\phi_i$ variables can be written as a function of $\cos\phi_1$ and $\cos\phi_N$. Thus using the above equation and with the help of the constraint of (8), we can reduce the $N-1$ equations of Theorem 1 to the following system of two equations

$$\begin{aligned} &m_z \|\mu\| \frac{\cos\theta_1 - \cos\theta_N \frac{\cos\phi_1}{\cos\phi_N}}{(\lambda_1 - \lambda_N)\cos\phi_1} + \frac{m_z^2}{2\sigma_z^2} \left( \zeta\left(m_z^2, \sigma_z^2, P\right) - 1 \right) = -1 \\ &\cos^2\phi_1 + \cos^2\phi_N + (\lambda_1 - \lambda_N)^2 \cos^2\phi_1 \cos^2\phi_N \times \\ &\sum_{i=2}^{N-1} \frac{\cos^2\theta_i}{\left[(\lambda_i - \lambda_N)\cos\theta_1 \cos\phi_N - (\lambda_i - \lambda_1)\cos\phi_1 \cos\theta_N\right]^2} = 1 \end{aligned} \tag{16}$$

where $m_z, \sigma_z$ are expressed as functions of $\cos\phi_1$, $\cos\phi_N$ by combining (9) and (10) with (15). Obviously Corollaries 1 and 2 for low and high power regimes can be simplified in the same way by using the second equation of (16) and only the first equation of (13) and (14) respectively.

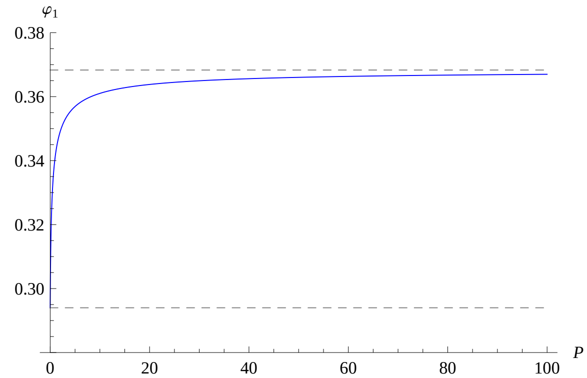


Fig. 1. Optimal solution of (16) as a function of power. Here we have used $\boldsymbol{\mu} = \|\boldsymbol{\mu}\| \{1, 1\}$, $\|\boldsymbol{\mu}\| = 2$ and $|k_{12}| = 0.5$. The lower dashed line corresponds to $\underline{\phi}$ and the upper dashed line to $\overline{\phi}$.

We solve the system of equations (16), using secant method's root finding algorithm. The secant method is practically an approximation of Newton's method. In order to calculate the derivative of (16) which is necessary for Newton's method, we use the secant method (approximate the derivative using the slope of a line through two points on the function, derivatives of (16) are continuous). Both algorithms can converge remarkably quickly, especially when the iteration begins in the vicinity of the desired root. Since the dominant eigenvector of $\boldsymbol{\mu}\boldsymbol{\mu}^H + \mathbf{K}$ is the sub-optimal solution which maximizes *SNR*, we know that it is close to the solution of (16). Thus, by beginning the iteration from this solution, the algorithm converges very fast. Our simulations showed that with maximum five iterations, we achieve a more than three digit precision which is sufficient for our calculations.

The most practical and efficient algorithm for the full rank optimization problem was presented recently in [6]. If beamforming is the optimal transmit strategy, solving equation (23) of that paper can track the beamforming vector. However, (23) is a matrix equation (system of $N$ equations) with complex elements and integrals. Although the Multidimensional Complex Iterative Algorithm in [6] provides the solution, our proposed solution is incomparably easier since (16) is only a system of two real valued equations. Moreover, in cases where beamforming is the optimal transmit strategy, our proposed scheme along with *SNR* optimization are the only practical solutions.

## V. Numerical results

In this section we present numerical results that give insight to the problem. We use as a toy case the $2 \times 1$ MISO system and a channel with $\boldsymbol{\mu} = \{\mu_1, \mu_2\}$ and $\mathbf{K} = \{k_{ij}\}_{\substack{i=1,2\\j=1,2}}$ where $k_{11} = k_{22} = 1$, $k_{12} = \frac{|k_{12}|}{\sqrt{2}} + \frac{|k_{12}|}{\sqrt{2}}\mathrm{i}$ and $k_{21} = k_{12}^*$ ($k_{12}^*$ is the conjugate of $k_{12}$). First, in Figure 1 we show the dependence of the optimum point on the available power. On the same Figure, we plot the solutions of low power and high power cases. The solution for any $P$ is always bounded by these

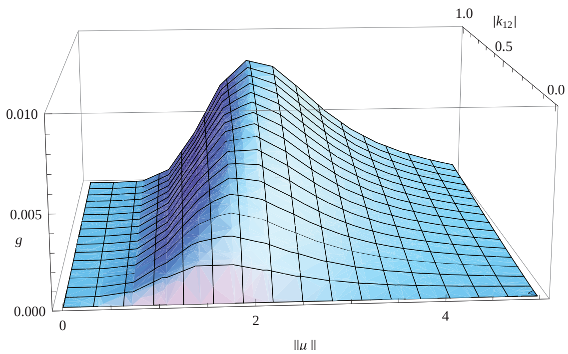


Fig. 2. Gain $g$ as a function of $\|\boldsymbol{\mu}\|$ and $|k_{12}|$. Here we have used $P = 10$, $\boldsymbol{\mu} = \|\boldsymbol{\mu}\| \{1 + \mathbb{i}, 1 + \mathbb{i}\}$ which yields $\theta_1 = 22.5^o$.

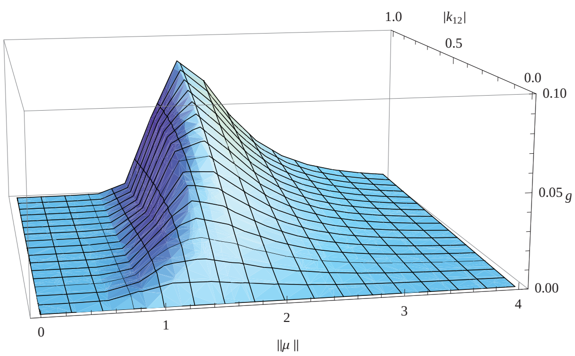


Fig. 3. Gain $g$ as a function of $\|\boldsymbol{\mu}\|$ and $|k_{12}|$. Here we have used $P = 10$, $\boldsymbol{\mu} = \|\boldsymbol{\mu}\| \{1 + \mathbb{i}, 2 - \mathbb{i}\}$ which yields $\theta_1 = 57^o$.

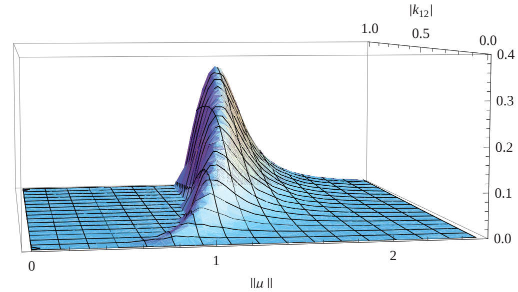


Fig. 4. Gain $g$ as a function of $\|\boldsymbol{\mu}\|$ and $|k_{12}|$. Here we have used $P = 10$, $\boldsymbol{\mu} = \|\boldsymbol{\mu}\| \{0.3, -0.3 - 0.3\mathbb{i}\}$ which yields $\theta_1 = 80^o$.

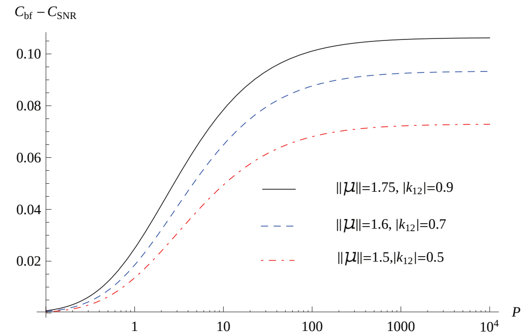


Fig. 5. Difference $C_{bf} - C_{SNR}$ as a function of transmitting power $P$ (the axis of $P$ is in logarithmic scale). Here we have used $\boldsymbol{\mu} = \|\boldsymbol{\mu}\| \{1+\mathbb{i}, 2-\mathbb{i}\}$ and different cases for the norm $\|\boldsymbol{\mu}\|$ and correlation $|k_{12}|$.

two extreme cases. Since these two values are close to each other, there exist settings where a good approximation can be obtained either by using the lower bound (which is given in closed form) or by using the upper bound which is more accurate for a wider span of values.

In the second numerical case, we examine the benefit of solving (16) compared to using the approach in [23] for maximizing *SNR*. In particular, we assign to $\mathbf{v}$ the dominant eigenvector of $\boldsymbol{\mu}\boldsymbol{\mu}^H + K$ and calculate the arising capacity, denoted as $C_{SNR}$. Finally, we calculate the gain $g \doteq C_{bf} - C_{SNR}$ and we plot it in Figures 2,3 and 4 for several settings.

First, note that for $\|\boldsymbol{\mu}\| = 0$ or $|k_{12}| = 0$ *SNR* optimization achieves the maximum capacity in all three cases. Next, the maximum gain is different for the three cases. Specifically, the gain is larger when the angle of the mean vector $\boldsymbol{\mu}$ with $\mathbf{u}_K^1$ ($\theta_1$) is closer to $90^o$ ($0.4bps$/Hz for $80^o$, which is significant). As $\theta_1$ increases, the difference between the dominant eigenvalue and the other eigenvalue of $\boldsymbol{\mu}\boldsymbol{\mu}^H + K$ becomes smaller. This implies that transmitting only over the dominant eigenvector becomes worse and worse. Thus the gain $g$ increases.

Next we study the efficiency of Capacity maximization versus *SNR* maximization by varying the available transmitting power for different cases of norm and correlation. Figure 5 verifies that for small power, the *SNR* solution is optimal $C_{bf} = C_{SNR}$. Also it gives an estimate of the actual capacity benefit for several values of transmitting power. Note that for high enough power, the gap between capacity optimization and *SNR* optimization is constant and depending on the parameters.Thus, it makes sense to compare the *SNR* solution with the high power regime solution $\overline{\phi}$.

Let $C_{\overline{\phi}}$ be the capacity achieved using the solution from (14). Figure 6 and Figure 7 show the differences $C_{\overline{\phi}} - C_{SNR}$ and $C_{bf} - C_{\overline{\phi}}$ correspondingly, where $P$ axis is in logarithmic scale. In Figure 6 we observe, that for $P > 1$, the solution of (14) outperforms the *SNR* maximization solution and for extremely large values of $P$, the gain between $C_{bf} - C_{SNR}$ is constant depending on the parameters. Also from Figure 7 we observe, that for $P > 10$, $C_{\overline{\phi}}$ is very close to the optimal solution of (16). For large values of $P$, $C_{bf}$ and $C_{\overline{\phi}}$ are equal, as expected. Therefore, for practical values of transmitting power $1 \leq P \leq 1000$, the solution of (14) outperforms the solution of [23] (or the solution of (13)). Also, for $P > 10$, $C_{\overline{\phi}}$ arising from (14), is practically equal to $C_{bf}$, so we can call it near-optimal for this range.

Figures 8 and 9 show the difference $C_{\overline{\phi}} - C_{SNR}$ as a function of the channel mean norm and power respectively for $10 \times 1$ systems. These Figures present similar results and are in accordance to Figures 2,3,4,5 revealing that, in general, an $N \times 1$ system behaves analogously to a $2 \times 1$ system.

Concluding the results we can infer an engineering rule of thumb for using the several available approaches for calculating an efficient beamforming vector $\mathbf{v}$. If the channel mean norm is large ($\|\boldsymbol{\mu}\| > 5$) or close to zero ($\|\boldsymbol{\mu}\| < 1$) the gain $g$ between the optimal solution and the *SNR* maximization solution is small and the *SNR* maximization method is preferred (solution of [23] or equivalently low power regime, (13)). For

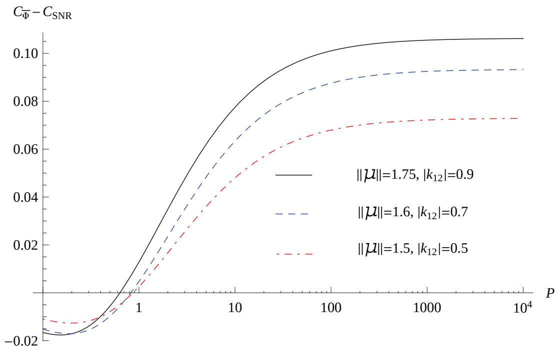


Fig. 6. Difference $C_{\overline{\phi}} - C_{SNR}$ as a function of transmitting power $P$ (the axis of $P$ is in logarithmic scale). Here we have used $\boldsymbol{\mu} = \|\boldsymbol{\mu}\| \{1+\mathbb{i}, 2-\mathbb{i}\}$ and different cases for the norm $\|\boldsymbol{\mu}\|$ and correlation $|k_{12}|$.

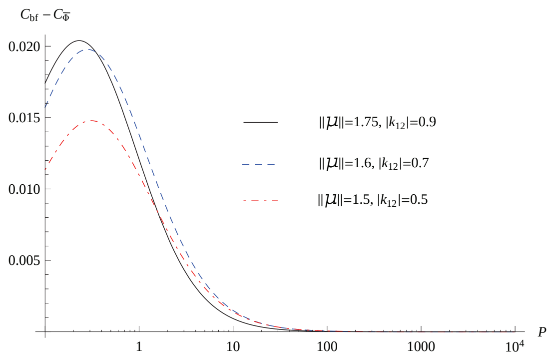


Fig. 7. Difference $C_{bf} - C_{\overline{\phi}}$ as a function of transmitting power $P$ (the axis of $P$ is in logarithmic scale). Here we have used $\boldsymbol{\mu} = \|\boldsymbol{\mu}\| \{1+\mathbb{i}, 2-\mathbb{i}\}$ and different cases for the norm $\|\boldsymbol{\mu}\|$ and correlation $|k_{12}|$.

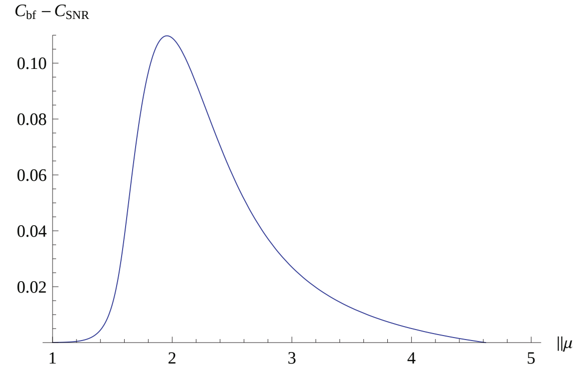


Fig. 8. Difference $C_{bf} - C_{SNR}$ as a function of channel mean norm for a $10 \times 1$ system. The parameters are $\theta_1 = \theta_6 = 85^o$, $\theta_2 = \theta_9 = \theta_{10} = 68^o$, $\theta_4 = \theta_5 = 63^o$, $\theta_7 = 74^o$, $\theta_8 = 80^o$, $P = 10$.

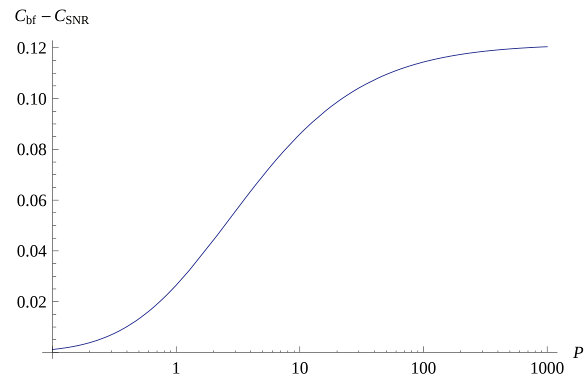


Fig. 9. Difference $C_{bf} - C_{SNR}$ as a function of transmitting power $P$ for the same $10 \times 1$ system as in Figure 8.

the intermediate values of norm $1 < \|\boldsymbol{\mu}\| < 5$, the solution of (16) should be used, unless $P > 10$ where the solution of (14) is equivalent and easier to compute.

## VI. Conclusions

We have investigated the capacity-achieving input covariance for an $N \times 1$ MISO system that uses beamforming as transmit strategy and the channel is known instantaneously at the receiver and statistically at the transmitter. Previously, there was no solution revealing the optimal beamforming vector, unless beamforming was the optimal transmit strategy (which occurs in specific cases, [20]). In this paper we presented an analytical approach which provides directly the optimal beamforming vector after solving numerically a system of two equations, (16). This analysis extremely simplifies the beamforming capacity optimization problem, outperforms the existing solutions and promises a similar solution for the full rank optimization problem. So far, *SNR* maximization is the simplest beamforming technique, which however provides sub-optimal results. Examining the $2 \times 1$ and $10 \times 1$ MISO systems indicatively, we have shown that the capacity achieved by *SNR* maximization may be up to $0.4 bps/Hz$ less than the optimal solution achieved by our proposed scheme for some cases. Due to the simplicity of *SNR* maximization method, we examined the comparison between the two schemes and presented a rule of thumb when to use each of the two approaches.

## Appendix A

*Proof of Theorem 1:* In the following we will focus on the partial derivative with respect to a particular variable (say $\phi_i$) and thus we will be using $(.)' \doteq \frac{\partial(.)}{\partial \phi_i}$. From (2) and (5) we have

$$C_{bf} = \int_0^\infty \log_2(1+Px^2)\frac{x}{\sigma_z^2} I_0\left(\frac{m_z}{\sigma_z^2}x\right) e^{-\frac{x^2+m_z^2}{2\sigma_z^2}} dx.$$

With a change of variables we obtain,

$$\begin{aligned} C_{bf} &= \int_0^\infty \log_2(1+Px)\frac{1}{2\sigma_z^2} I_0\left(\frac{m_z}{\sigma_z^2}\sqrt{x}\right) e^{-\frac{x+m_z^2}{2\sigma_z^2}} dx \\ &= \frac{1}{2\sigma_z^2} e^{-\frac{m_z^2}{2\sigma_z^2}} \int_0^\infty \log_2(1+Px) I_0\left(\frac{m_z}{\sigma_z^2}\sqrt{x}\right) e^{-\frac{x}{2\sigma_z^2}} dx \qquad (17) \\ &= \frac{1}{2\sigma_z^2} e^{-\frac{m_z^2}{2\sigma_z^2}} I, \end{aligned}$$

where $I = \int_0^\infty \log_2(1+Px) I_0\left(\frac{m_z}{\sigma_z^2}\sqrt{x}\right) e^{-\frac{x}{2\sigma_z^2}} dx$.

The necessary but not sufficient condition of $C_{bf}$ to have a local extremum is

$$\frac{\partial C_{bf}}{\partial \phi_1} = \frac{\partial C_{bf}}{\partial \phi_2} = \cdots = \frac{\partial C_{bf}}{\partial \phi_{N-1}} = 0. \qquad (18)$$

Thus from (17), (18) and $\sigma_z^2 > 0$ we have

$$\left(\frac{1}{2\sigma_z^2}e^{-\frac{m_z^2}{2\sigma_z^2}}\right)' I + \frac{1}{2\sigma_z^2}e^{-\frac{m_z^2}{2\sigma_z^2}} I' = 0 \Leftrightarrow$$
$$I' = I\left[\frac{\sigma_z^{2\prime}}{\sigma_z^2} + \left(\frac{m_z^2}{2\sigma_z^2}\right)'\right], \tag{19}$$

But, $I' = \int_0^\infty \log_2(1+Px)\left(I_0\left(\frac{m_z}{\sigma_z^2}\sqrt{x}\right)e^{-\frac{x}{2\sigma_z^2}}\right)' dx$ which after some mathematical manipulations with the derivative becomes

$$I' = \left(\frac{m_z}{\sigma_z^2}\right)' Z_1 - \left(\frac{1}{2\sigma_z^2}\right)' Z_0, \tag{20}$$

where $Z_i = \int_0^\infty \log_2(1+Px)\, x^{\frac{2-i}{2}} I_i\left(\frac{m_z}{\sigma_z^2}\sqrt{x}\right) e^{-\frac{x}{2\sigma_z^2}} dx$.

Expanding $I_i(x)$ in its infinite sum representation, i.e. $I_i(x) = \sum_{k=0}^\infty \frac{1}{k!(k+i)!}\left(\frac{x}{2}\right)^{2k+i}$, we can write $Z_i$ as

$$Z_i = \sum_{k=0}^\infty \frac{\left(\frac{m_z}{2\sigma_z^2}\right)^{2k+i}}{k!(k+i)!}\int_0^\infty \log_2(1+Px)\, x^{k+1} e^{-\frac{x}{2\sigma_z^2}} dx$$

and with the help of

$$\int_0^\infty \ln(1+x)\, x^{k-1} e^{-\beta x} dx = \Gamma(k)\, e^\beta \beta^{-k} \sum_{n=0}^{k-1} \beta^n \Gamma(-n,\beta)$$

from Appendix A in [25], we derive

$$Z_i = \frac{e^{\frac{1}{2\sigma_z^2 P}}}{\ln 2}\sum_{k=0}^\infty \frac{(k+1)!}{k!(k+i)!}\left(\frac{m_z}{2\sigma_z^2}\right)^{2k+i}(2\sigma_z^2)^{k+2} S_{k+1}, \tag{21}$$

where $S_k = \sum_{n=0}^k \left(2\sigma_z^2 P\right)^{-n}\Gamma\left(-n, \frac{1}{2\sigma_z^2 P}\right)$. Similarly we can solve the integral $I$ which becomes

$$I = 2\sigma_z^2 \frac{e^{\frac{1}{2\sigma_z^2 P}}}{\ln 2}\sum_{k=0}^\infty \frac{1}{k!}\left(\frac{m_z^2}{2\sigma_z^2}\right)^k S_k. \tag{22}$$

Substituting (20), (22) in (19) we get

$$\left(\frac{m_z}{\sigma_z^2}\right)' m_z \sum_{k=0}^\infty \frac{1}{k!}\left(\frac{m_z^2}{2\sigma_z^2}\right)^k S_{k+1} - \left(\frac{1}{\sigma_z^2}\right)' 2\sigma_z^2 \sum_{k=0}^\infty \frac{k+1}{k!}$$
$$\left(\frac{m_z^2}{2\sigma_z^2}\right)^k S_{k+1} = \left[\frac{(\sigma_z^2)'}{\sigma_z^2} + \left(\frac{m_z^2}{2\sigma_z^2}\right)'\right]\sum_{k=0}^\infty \frac{1}{k!}\left(\frac{m_z^2}{2\sigma_z^2}\right)^k S_k.$$

Noticing $S_{k+1} = (2\sigma_z^2 P)^{-k+1}\Gamma\left(-k-1, \frac{1}{2\sigma_z^2 P}\right) + S_k$ and after some mathematical manipulations, we finally get (11). ■

## Appendix B

*Proof of Theorem 2:* We begin by showing that $\left.\frac{\partial C_{bf}}{\partial \phi_i}\right|_{\phi_i=0} \geq 0$ and $\left.\frac{\partial C_{bf}}{\partial \phi_i}\right|_{\phi_i=\frac{\pi}{2}} \leq 0$, $\forall i = 1, \ldots, N-1$. From (8) and (9) we have that

$$\frac{\partial \sigma_z^2}{\partial \phi_i} = \frac{1}{2}(\lambda_i - \lambda_N)\cos\phi_i \sin\phi_i.$$

Then evidently

$$\left.\frac{\partial \sigma_z^2}{\partial \phi_i}\right|_{\phi_i=0} = \left.\frac{\partial \sigma_z^2}{\partial \phi_i}\right|_{\phi_i=\frac{\pi}{2}} = 0, \forall i = 1, \ldots, N-1, \tag{23}$$

where $(.)|_{\phi_i=0}$ denotes the value of $(.)$ at $\phi_i = 0$. From (19), where the first equation is equivalent to $\frac{\partial C_{bf}}{\partial \phi_i} = 0$, with some minor manipulations and utilizing (23) we have that

$$\begin{aligned}\left.\frac{\partial C_{bf}}{\partial \phi_i}\right|_{\phi_i=0} &\geq 0 \Leftrightarrow \\ \left.\frac{\partial I}{\partial \phi_i}\right|_{\phi_i=0} &\geq \left.\left(\frac{I}{\sigma_z^2}\frac{\partial m_z^2}{\partial \phi_i}\right)\right|_{\phi_i=0}.\end{aligned}$$

From (20) we have $\left.\frac{\partial I}{\partial \phi_i}\right|_{\phi_i=0} = \left.\left(\frac{Z_1}{\sigma_z^2}\frac{\partial m_z^2}{\partial \phi_i}\right)\right|_{\phi_i=0}$. Taking into account (21) and (22) we obtain

$$\left.\left(2m_z \frac{\partial m_z}{\partial \phi_i}\sum_{k=0}^\infty \frac{1}{k!}\left(\frac{m_z^2}{2\sigma_z^2}\right)^k S_{k+1}\right)\right|_{\phi_i=0} \geq \tag{24}$$
$$\left.\left(2m_z \frac{\partial m_z}{\partial \phi_i}\sum_{k=0}^\infty \frac{1}{k!}\left(\frac{m_z^2}{2\sigma_z^2}\right)^k S_k\right)\right|_{\phi_i=0}, \tag{25}$$

which is true since $m_z \geq 0$, $\left.\frac{\partial m_z}{\partial \phi_i}\right|_{\phi_i=0} \geq 0$ and $S_{k+1} \geq S_k$. Thus, we have shown the first inequality.

When $\phi = \frac{\pi}{2}$, the inequality (25) is inverted and $\left.\frac{\partial m_z}{\partial \phi_i}\right|_{\phi_i=0} \leq 0$ and thus similarly the second inequality is shown.

Consequently, each equation $\frac{\partial C_{bf}}{\partial \phi_i} = 0$ will have at least one solution in $[0, \frac{\pi}{2}]$ independently of the other variables $\boldsymbol{\phi}\backslash\{\phi_i\}$. This implies that the capacity of (5) has at least one local extremum and the system of non-linear equations defined in Theorem 1 has at least one solution. Also, due to the claim above, the maximum is not attained at the boundaries of the domain of $\boldsymbol{\phi}$ unless a stationary point is located there. Thus, the second claim of the theorem is proved. ■

## Appendix C

*Proof of Corollary 2:* Note that

$$\lim_{x\to 0}\frac{\Gamma(a,x)}{x^a} = -\frac{1}{a}.$$

Thus taking into account that $\sum_{k=0}^\infty \frac{x^k}{k!}\frac{1}{k+1} = \frac{e^x-1}{x}$ and $\sum_{k=0}^\infty \frac{x^k}{k!}\frac{1}{k+2} = \frac{(x-1)e^x-1}{x^2}$ we obtain

$$\lim_{P\to\infty}\zeta = \frac{\frac{1}{2\sigma_z^2 P}\sum_{k=0}^\infty \frac{\left(\frac{m_z^2}{4\sigma_z^4 P}\right)^k}{k!}\frac{(2\sigma_z^2 P)^{k+2}}{k+2}}{\sum_{k=0}^\infty \frac{\left(\frac{m_z^2}{4\sigma_z^4 P}\right)^k}{k!}\frac{(2\sigma_z^2 P)^{k+1}}{k+1}} \tag{26}$$
$$= \frac{\left(\frac{m_z^2}{2\sigma_z^2}-1\right)e^{\frac{m_z^2}{2\sigma_z^2}}+1}{\left(e^{\frac{m_z^2}{2\sigma_z^2}}-1\right)\frac{m_z^2}{2\sigma_z^2}}. \tag{27}$$

Finally, substituting (27) in (11) and after some manipulations we obtain the result of Corollary 2. ■

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